Rayleigh-Taylor instability under a flat beam

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A theoretical and experimental study on the Rayleigh-Taylor instability of a thin liquid film is presented. The experiments were performed by coating the underside of a flat beam with uniform liquid film. The size of the beam is such that a linear arrangement of drops is formed allowing the measurement of the drops growth. The experimental growth of the drops has been compared to the results obtained from a stability analysis performed on the equation that controls the thickness evolution and an excellent agreement is found. In addition, both time and length scales of the phenomenon are established.

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I. INTRODUCTION

The study of the flow of thin liquid films is of great interest for basic physics as well as for industrial applications such as painting and coating processes. When the film is subjected to a reverse density gradient such as occurs when it coats the underneath of a solid surface, Rayleigh-Taylor instability [1–6] appears in the form of drops. Usually, the drops arrange in nonregular patterns [6] and the two dimensions of the film must be taken into account.

However, in very special conditions such as the original experiment presented here, it is possible to analyze the problem in one dimension in order to simplify the analysis without loss of generality.

This work is intended to be a simple approach to the Rayleigh-Taylor instability in thin films. That is, considering a one-dimensional film of constant volume, conditions not fulfilled in Refs. [2–6]. The key idea of the experiment is to obtain a linear alignment of drops in order to study their growth as a function of time. This is achieved by coating the lower face of a beam of the appropriate width with a uniform film of liquid which will become unstable. An explicit evolution equation of the film thickness h, simplified by using the lubrication approximation [7] is used.

The paper is organized as follows: in Sec. II the experimental setup is presented, in Sec. III the evolution equation for the film is analyzed together with a linear stability analysis. In Sec. IV the experimental results are shown and, finally, Sec. V is devoted to analyze the results.

II. EXPERIMENTAL PROCEDURE

The experimental setup is shown schematically in Fig. 1. The flat beam is built in polished aluminum and has a length $l=16$ cm and a width $w=1.2$ cm. The length and the width were chosen in order to obtain a linear arrangement of about ten drops, as shown in Fig. 2. As mentioned above, this is the key feature of the experiment and this topic will be discussed in Sec. IV.

The beam is fixed to the frame and fully coated with a liquid film of uniform thicknesses of 150 and 350 μm using a bladelike [8] device with the wetted surface facing upside. The film is left a reasonable time in order to allow the slight irregularities to vanish and then is turned to its final position to let the experiment begin. The aluminum rectangular frame is used to turn upside down the beam after the film is deposited and the base is made in iron which gives the necessary mechanical stability to the whole system.

The fluid used in the experiments is polydimethylsiloxane (silicon oil) with viscosity $\mu=1000$ cP, density $\rho=0.97$ g/cm^3, surface tension $\gamma=22.7$ mN/m, and very suitable characteristics: it wets completely the surface and it is nonvolatile. The former ensures that no dry spots will appear and the latter keeps constant the mass of the fluid and avoids the apparition of surface-tension gradients.

With a charge-coupled device camera a side view of the phenomenon is recorded. The ulterior image treatment allows us to determine the "wavelength," mean distance between drops, and depending on the initial film thickness, the growth rate of the drops.

An optical displacement sensor (ODS), from Acuity Research, model No AR-600–0125 [9], is used to measure the film thickness along the beam as a function of time. It is displaced automatically and takes about 15 s to travel the beam, this time is small compared to the characteristic time of the experiment [7] and so, the profile can be considered to be obtained instantaneously. In each path the ODS takes around 2000 measurements along the central axis, x which is enough to obtain a precise profile $h(x)$.

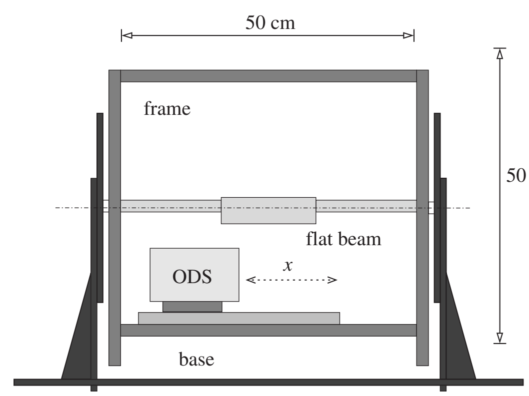

FIG. 1. Schematic view of the experimental setup used to obtain Rayleigh-Taylor instability under a flat beam.

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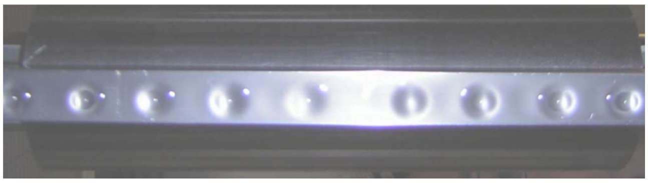

FIG. 2. (Color online) Photography of the drops aligned along the beam.

FIG. 3. Drop formation during an experiment with 150 μm of initial thickness. Time difference between images is ~1200 s.

Since the ODS measures distances, it is necessary to determine first $d_0(x)$, distance to the dry beam surface (background value) and then $d(x)$: distance to the wet beam surface corrected by refraction, so $h(x)=d_0(x)-d(x)$. With this procedure and a good enough mechanical stability in order to keep constant in time the distance between the sensor and the beam, an indeterminacy of ± 7 μm in the value of the local thickness is obtained. During each experiment from 30 to 50 profiles are obtained, so, the appropriate interval for the analysis between consecutive profiles ranges from 30 s to several minutes.

III. THICKNESS EVOLUTION EQUATION

The equation that controls the temporal evolution of the thickness using the lubrication approximation has been obtained in previous works, such as that of the reference [6]. For our purposes, it is enough to rewrite it at higher order as

$$\frac{\partial h}{\partial t} = -\frac{gh^3}{3\nu}[\nabla^2 h + \kappa^2 \nabla^4 h], \tag{1}$$

where g is the gravity, ν the kinematic viscosity, $\kappa=\sqrt{\gamma/\rho g}$ the capillary length and ∇ is the two-dimensional (2D) operator, $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$.

One-dimension linear stability analysis

In order to obtain some of the characteristics of the instability that appears in the suspended film a linear stability analysis was performed on the one-dimension version of Eq. (1),

$$\frac{\partial h}{\partial t} = -\frac{gh^3}{3\nu}\left[\frac{\partial^2 h}{\partial x^2} + \kappa^2 \frac{\partial^4 h}{\partial x^4}\right], \tag{2}$$

by perturbing the *stable* solution $h(x,t)$ [10],

$$h^p(x,t) = h(x,t) + \psi_0 e^{iq_x x} e^{\sigma t}, \tag{3}$$

where σ can be taken as real and $\psi_0/h \ll 1$. The sign of σ determines if the perturbation will grow ($\sigma>0$) or decay ($\sigma<0$) with time.

Replacing Eq. (3) into Eq. (2) and keeping only linear terms on ψ_0, the following dispersion relation is obtained:

$$\sigma = \frac{gh_0^3}{3\nu} q_x^2 (1-\kappa^2 q_x^2), \tag{4}$$

which is positive (i.e., unstable film) for $0<q_x<\kappa^{-1}$. The maximum of Eq. (4) is obtained for

$$\lambda_x^{max} = \frac{2\pi}{q_x^{max}} = 2\pi\sqrt{2}\sqrt{\frac{\gamma}{\rho g}}, \tag{5}$$

which gives the maximum growth rate,

$$\sigma^{max} = \frac{1}{12}\frac{g^2 h_0^3 \rho}{\nu\gamma}. \tag{6}$$

Both expressions are identical to those obtained for the Rayleigh-Taylor instability in infinite flat plates [1,6], that is, in a two-dimensional configuration.

IV. EXPERIMENTAL RESULTS

In a first stage 2D experiments were performed in order to determine the characteristic time and lengths of the system (liquid solid). It was found that suitable thicknesses with reasonable evolution time and no dripping range between 100 and 400 μm.

In addition, it was observed (as in Ref. [6]) the formation of a regular pattern of drops with a typical size of 8 and 15 mm between two neighbor maxima. So, the beam length and width, 160 and 12 mm, respectively, were chosen in order to force the formation of a line of centered and complete drops along the beam axis (see Fig. 2). The drops grow in different locations in each experiment, which shows the small influence of the substrate roughness on the film evolution.

In Fig. 3 a sequence of images of the drop formation process during an experiment with initial film thickness h_0 =150 μm is shown and Fig. 4 displays two typical profiles for the same initial thickness. Figure 4(a) shows the initial profile where the average value of 150 μm can be observed.

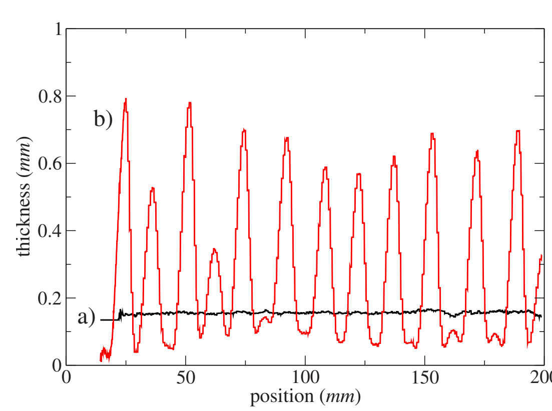

FIG. 4. (Color online) Two profiles obtained in an experiment with an initial thickness of 150 μm at (a) $t\sim 0$ and (b) $t\sim 1000$ s.

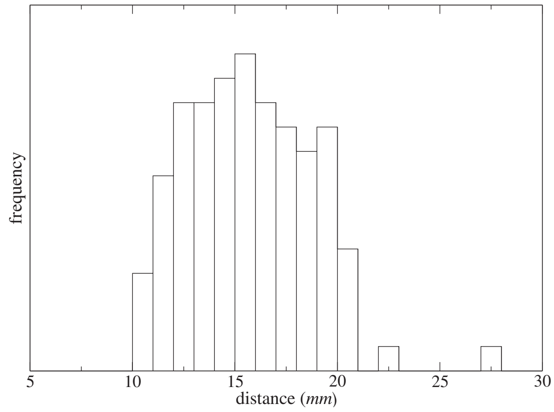

FIG. 5. Distribution of distances between drops (wavelength) corresponding to experiments performed with both initial thicknesses: 150 and 350 μm.

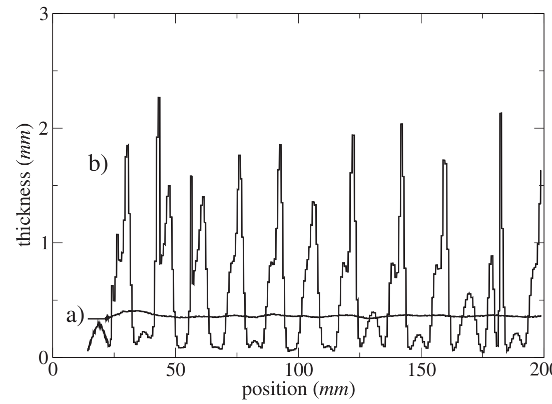

FIG. 7. Two profiles obtained in an experiment with an initial thickness of 350 μm at (a) $t \sim 0$ and (b) $t \sim 350$ s.

Profile (b), obtained at $t = 1000$ s, shows a developed instability.

The drops with a maximum thickness, h_{max}, of about 700 μm are distributed in an approximately regular array connected with zones of minimum thickness h_{min}. The distribution of the distances between consecutive maxima collected from several experiments and using both experimental techniques (images and profile) is shown in Fig. 5 showing a wavelength $\lambda \sim 15$ mm.

When the initial thickness is increased two expected differences are observed: the characteristic times are shorter and drops amplitudes are larger (Fig. 6). In Fig. 7 typical film profiles are shown for experiments with $h_0 = 350$ μm. It can be seen that when completely developed drops height is $h_{max} \sim 2100$ μm and that occurs for $t \sim 400$ s while in the previous case the height was three times smaller and the time, five times larger. On the contrary, it was found that λ remains close to the previous values as shown in Fig. 5 where experimental data corresponding to both h_0 values are superimposed. The amplitude of every drop in the profile at time t is measured and the mean value $\Delta h(t)$ is obtained.

In Fig. 8 the variation in Δh with time t is plotted for different experiments with initial thickness 150 μm (empty symbols) and $h_0 = 350$ μm (black symbols). At short times an exponential growth is observed in both cases while at longer times ($t \gtrsim 2000$ s and $t \gtrsim 400$ s, respectively) the drops stop growing and attain a constant height. This behavior, due to the small volume of liquid left in the film connecting the droplets, is observed for the initial thicknesses smaller than 500 μm, while dripping appears for larger h_0. This result is in agreement with a theoretical approach [11] which predicts that the maximum volume of a drop only supported by surface tension is $\sim 20\kappa^3$.

FIG. 6. Drop formation during an experiment with 350 μm of initial thickness. Time difference between images is ~ 100 s.

V. ANALYSIS AND DISCUSSION

As explained in the introduction this work has been designed to obtain simple experimental results of Rayleigh-Taylor instabilities in thin films. The key feature of the experiment is to produce a linear arrangement of drops by using an original variation in the flat plate with which it shares many characteristics such as the constancy of the film volume and the evolution equation. The configuration allows to follow the growth of every drop by measuring the height as a function of time.

The stability analysis performed on Eq. (2) has shown that there is a range of wavelengths where instabilities can develop $\lambda_x > 2\pi\sqrt{\frac{\gamma}{\rho g}}$. This result is coherent with the origin of the Rayleigh-Taylor instability as a competition between gravity and surface tension, which avoids short wavelengths.

The wavelengths experimental results, Fig. 5, show that there is a narrow distribution around a value close to that given by Eq. (5) which, in our experimental conditions is $\lambda_x^{max} \sim 13.5$ mm. Although there are some few drops separated by longer distances, no drops closer than 10 mm have been observed.

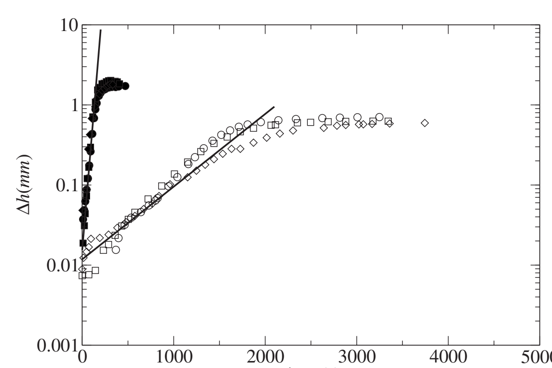

FIG. 8. Mean amplitude of the drops as a function of time for experiments performed with both initial thicknesses: 150 (empty symbols) and 350 μm. The straight lines show the data mean slope.

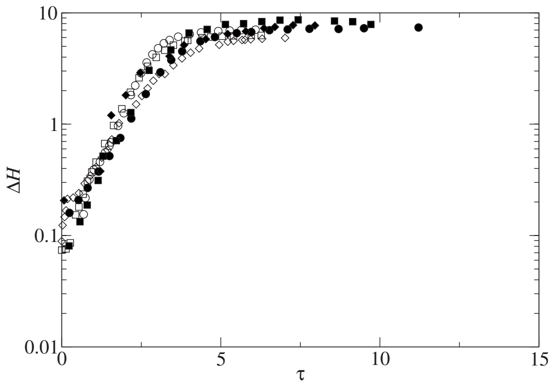

FIG. 9. Dimensionless form of Fig. 8 obtained by using σ_0^{-1} and h_0 as time and thickness scales, respectively.

At short times, as shown in Fig. 8, the heights of the drops grow exponentially. This fact allows to obtain two important results: a quantitative meaning for *short time* as used in the linear stability analysis, and the value of the corresponding growth rate, σ. From data in Fig. 8 a mean experimental value of $\sigma_{150}=2\times10^{-3}\ \mathrm{s}^{-1}$ is obtained. Replacing this value in Eq. (6) a thickness of 185 μm is obtained, which is close to the 150 μm thickness of the experiment. In a similar way, $\sigma_{350}=2\times10^{-2}\ \mathrm{s}^{-1}$ is the experimental mean value obtained for data corresponding to 350 μm of initial thickness and through Eq. (6) $h_0=385\ \mu$m is found, in excellent agreement with the experimental thickness.

Both results show that Eq. (2) is a good approximation to the drops growth and allows to assume that $\sigma^{-1}=\frac{12\nu\gamma}{g^2 h_0^3\rho}$ could be a good time scale of the phenomenon for an initial thickness h_0 while the obvious scale for the thickness is h_0. When replotting Fig. 8 in dimensionless form, Fig. 9, all the curves collapse into an universal curve, showing that the drops amplitude reach a maximum value of approximately 10 times the initial thickness at $t\sim5\sigma^{-1}$. A theoretical approach about this behavior is under development.

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